

# **Indifferentiability of Merkle-Damgård Hash Function Revisited: Impact to Practical Cryptosystems**

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(Joint work with Naito, Yoneyama, Ohta)

# Indifferentiability of Merkle-Damgård Hash Function **Revisited** Impact to Practical Cryptosystems

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**versus Private-interface-leaking  
Random Oracle**

# **Indifferentiability of Merkle-Damgård Hash Function Revisited: Impact to Practical Cryptosystems**

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- **Security of individual protocol using MD:  
Unclear (instead of insecure)!**
- **Protocols in Weakened Random Oracle:  
Continuously studied!**

# Outline

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- Background
- Our Goal
- Private-interface-leaking Random Oracles
- Conclusion

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# A Bad Fact

**Any Dedicated Hash Function  
can be easily distinguished from  
a Random Oracle**

*Canetti, Goldreich and Halevi, “The Random Oracle Methodology, Revisited”, STOC 1998.*

## Countermeasure?

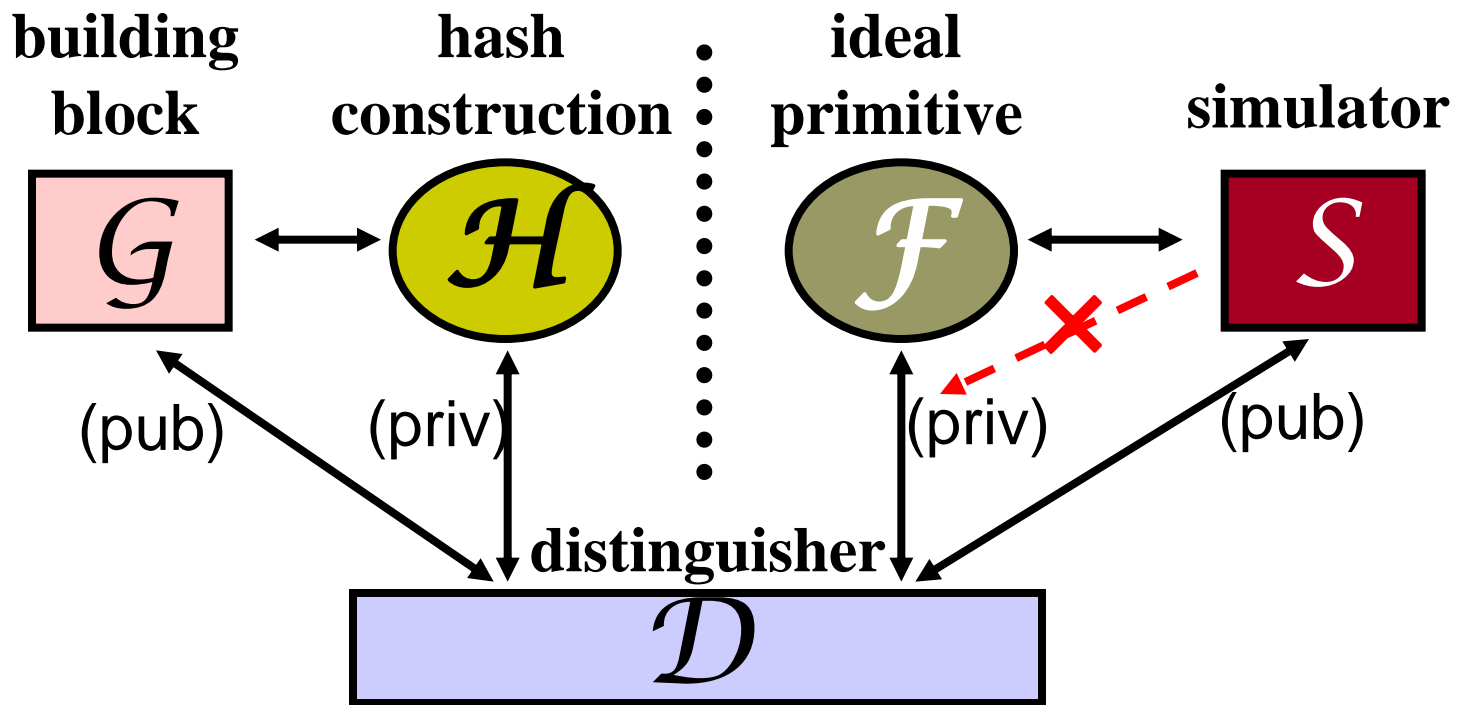
# Indifferentiability!!!

- **General Applications: Maurer *et al.***
- **Hash Function: Coron *et al.***

*Maurer, Renner and Holenstein, “Indifferentiability, Impossibility Results on Reductions, and Applications to the Random Oracle Methodology ”, TCC 2004.*

*Coron, Dodis, Malinaud and Puniya, “Merkle-Damgård Revisited: How to Construct a Hash Function”, CRYPTO2005*

# Application to Hash Function

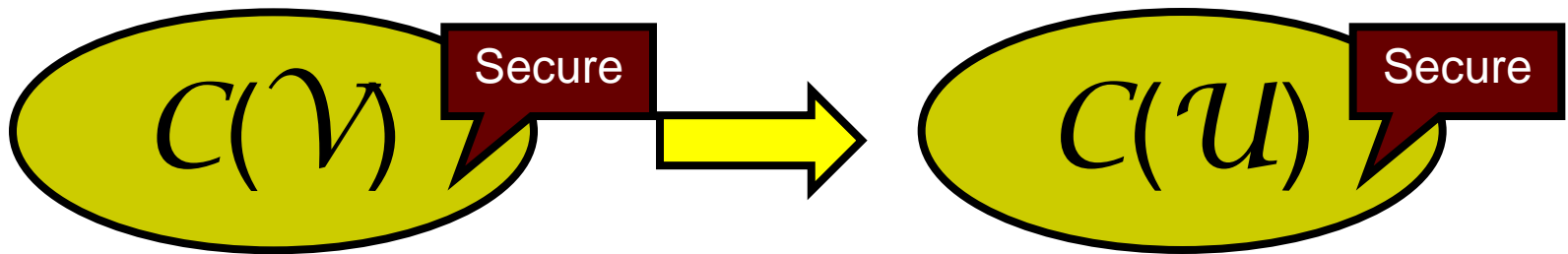


$$|\Pr[\mathcal{D}(\mathcal{H}, G) = 1] - \Pr[\mathcal{D}(\mathcal{F}, S) = 1]| < \text{negl. iff } \mathcal{H} \sqsubseteq \mathcal{F}$$



# Composition Theorem for Cryptosystems

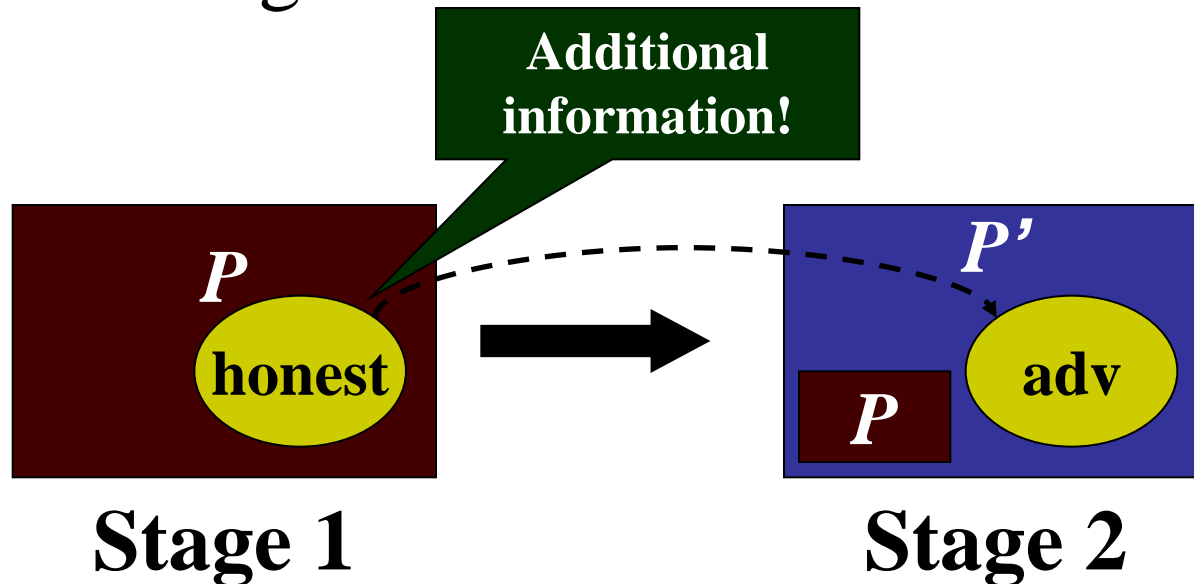
- If a Primitive  $U$  is indifferentiable from a Primitive  $V$ , ( $U \sqsubseteq V$ ), for any secure cryptosystem  $C(V)$ ,  $C(U)$  is also secure.



$$C(U) > C(V)$$

# A Remark

- Only for Single Stage
- Multi-Stage: **Reset Indifferentiability!!!**



*Ristenpart, Shacham and Shrimpton, "Careful with Composition: Limitations of the Indifferentiability Framework", EUROCRYPT2011.*

# A Remark

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- Only for Single Stage
- Multi-Stage

**This talk deals with cryptosystems  
with only a single stage!**



# A Bad Fact

**Merkle-Damgård Hash Function  
is not indifferentiable from a  
Random Oracle**

*Coron, Dodis, Malinaud and Puniya, “Merkle-Damgård Revisited: How to Construct a Hash Function”, CRYPTO2005*

# The Consequence

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- MD is most popular hash function mode.
- The security of Cryptosystems using popular hash functions becomes **unclear**, even in the ideal model.

## Countermeasure?

# Repair MD!!!

- Tailor the last block operation
- Tailor the message padding algorithm

# Actually Cryptographers did more!

## Sufficient properties to extend domain of an ideal primitive

- Pre-image Awareness
- Computable Message Awareness

*Dodis, Ristenpart and Shrimpton, “Salvaging Merkle-Damgård for Practical Applications”, EUROCRYPT2009.*

*Bhattacharyya, Mandal and Nandi, “Security Analysis of the Mode of JH Hash Function”, FSE2011*

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# Shall we give up MD completely?

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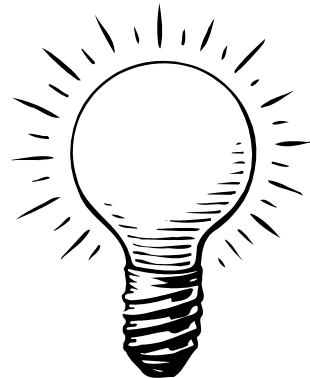
- Many popular hash functions are in MD mode, say SHA-2.
- The impact to cryptosystems is not clear yet.

**Goal: make it clear!!!**

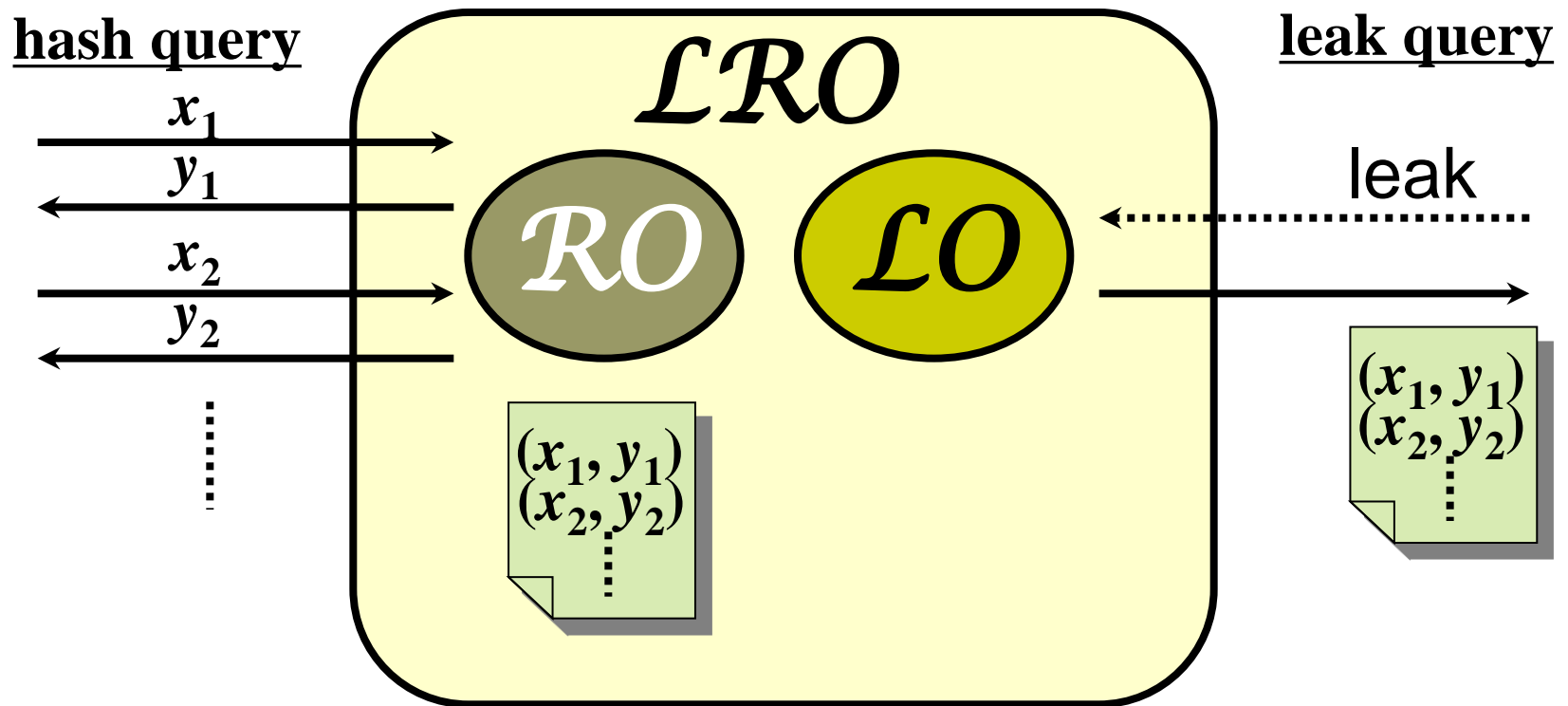
# How?

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**Study of cryptosystems in  
Weakened Random Oracle  
inspired us!**

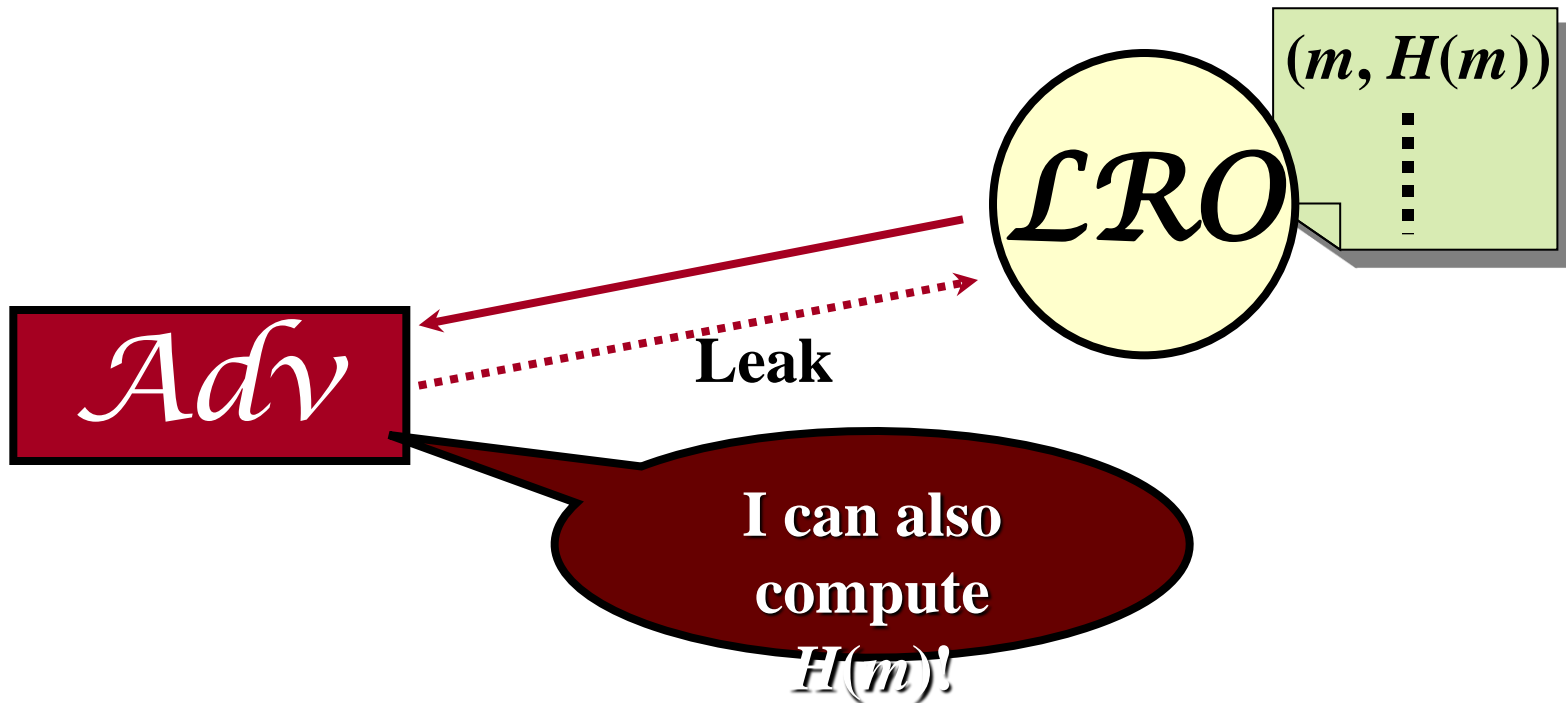


# Example: Leaky Random Oracle (LRO)



Yoneyama, Miyagawa and Ohta, "Leaky Random Oracle", ProvSec2008

# Full-domain Hash Signature in LRO



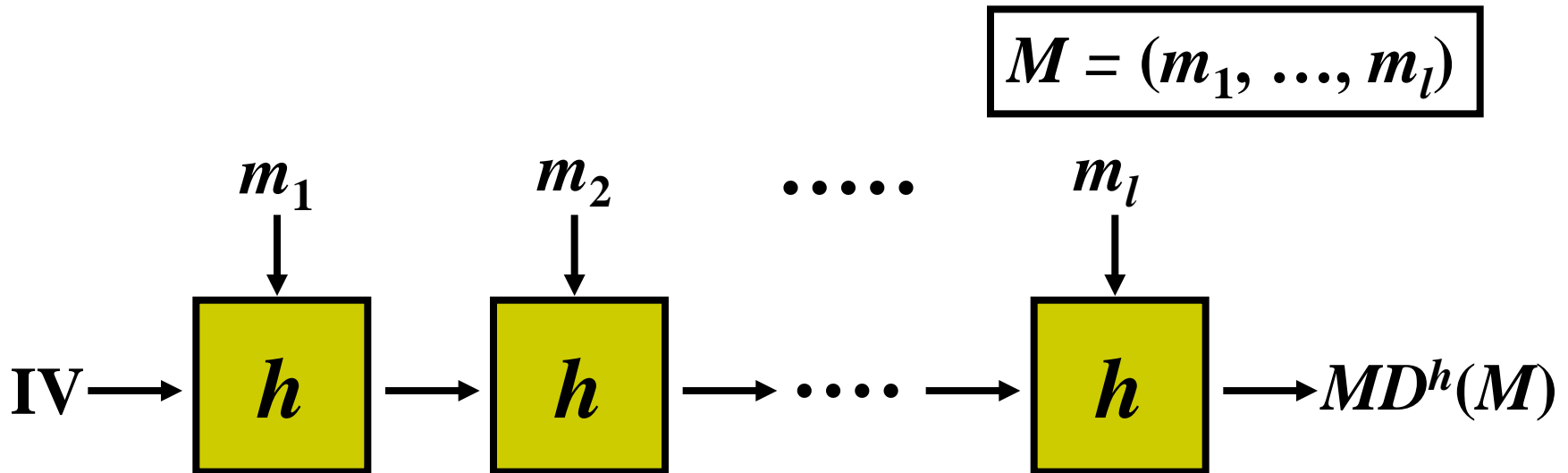
Intuitively,  $\{(m, H(m))\}$  is **not secret** to adversary.

Moreover, we proved that MD is **indifferentiable** from LRO.



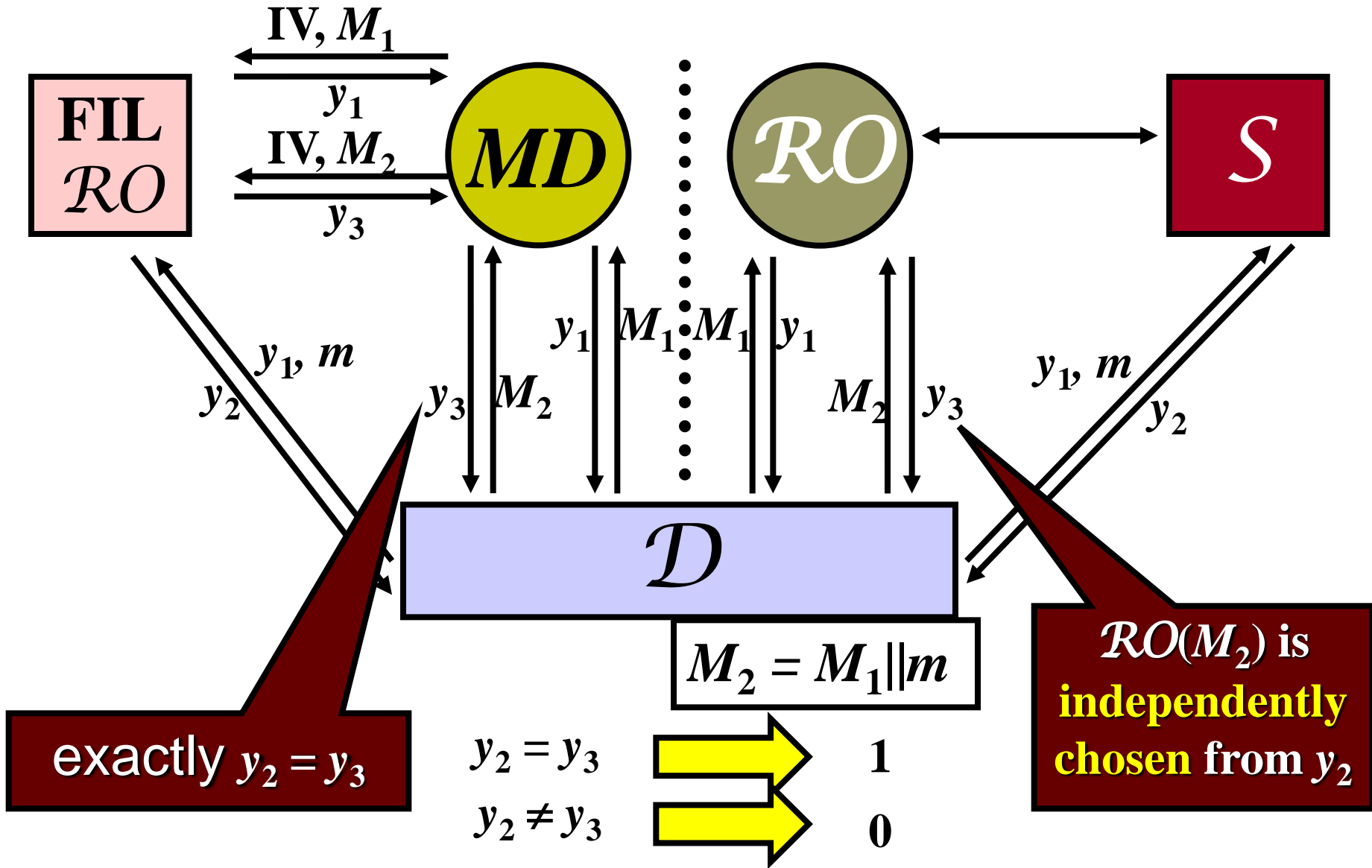
**FDH (actually many Digital Signature Schemes) is secure in MD mode.**

# MD Mode

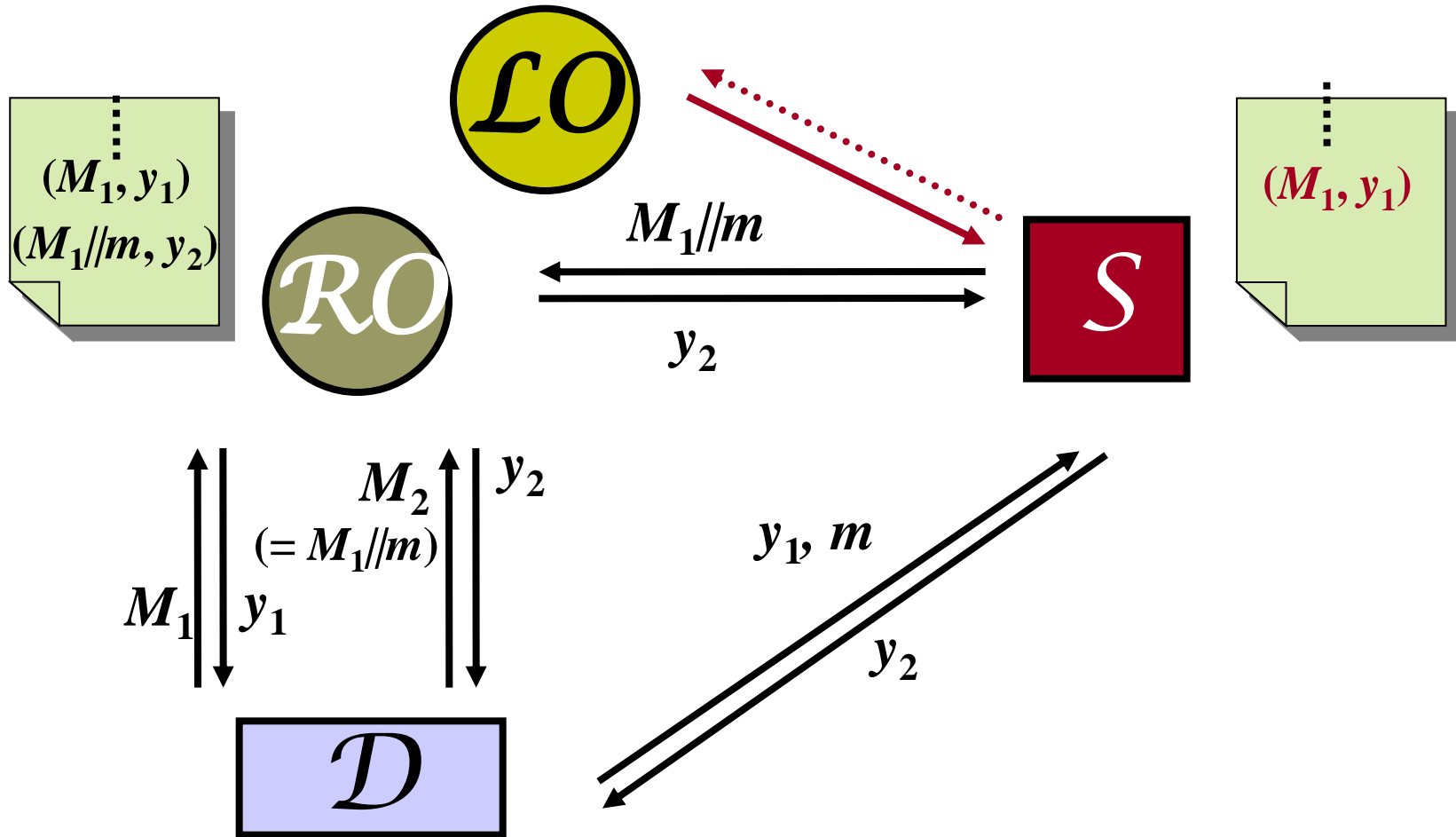


Fixed-input-length  
random oracle

**Length Extension Attack (LEA)** can distinguish it from RO.



# Intuition of MD $\sqsubseteq$ LRO





# Modular Approach

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- Define **private-interface-leaking** random oracles  $\tilde{RO}$ :  $MD \sqsubseteq \tilde{RO}$ .
- **Re-evaluate** the security of **practical cryptosystems** in  $\tilde{RO}$ .

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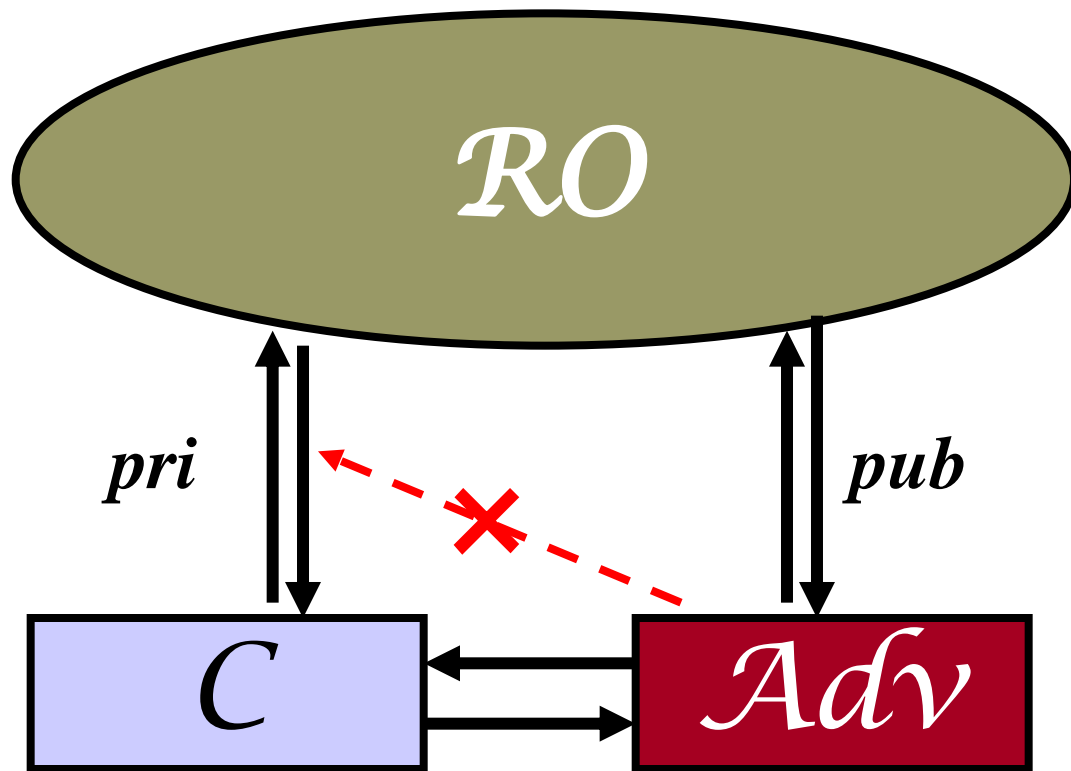
# Leaky Random Oracle

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- Independent work by Dodis *et al.*: **Public-use Random Oracle**
- **Secure**: FDH, Fiat-Shamir Signature, ...
- **Insecure**: OAEP, RSA-KEM...
  - Too much information is leaked.

*Dodis, Ristenpart and Shrimpton, "Salvaging Merkle-Damgård for Practical Applications", EUROCRYPT2009.*

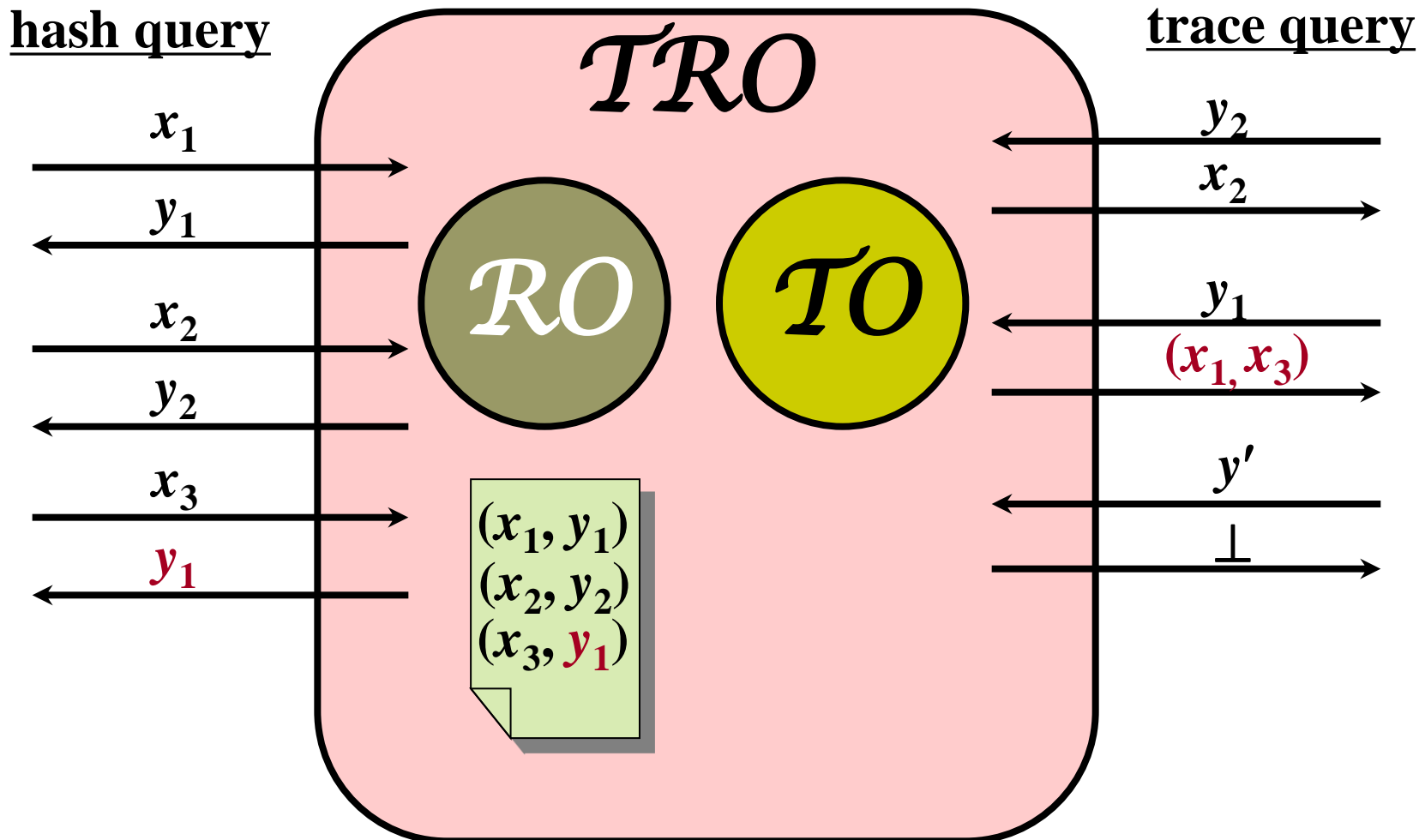
# LRO Leaks too much Information



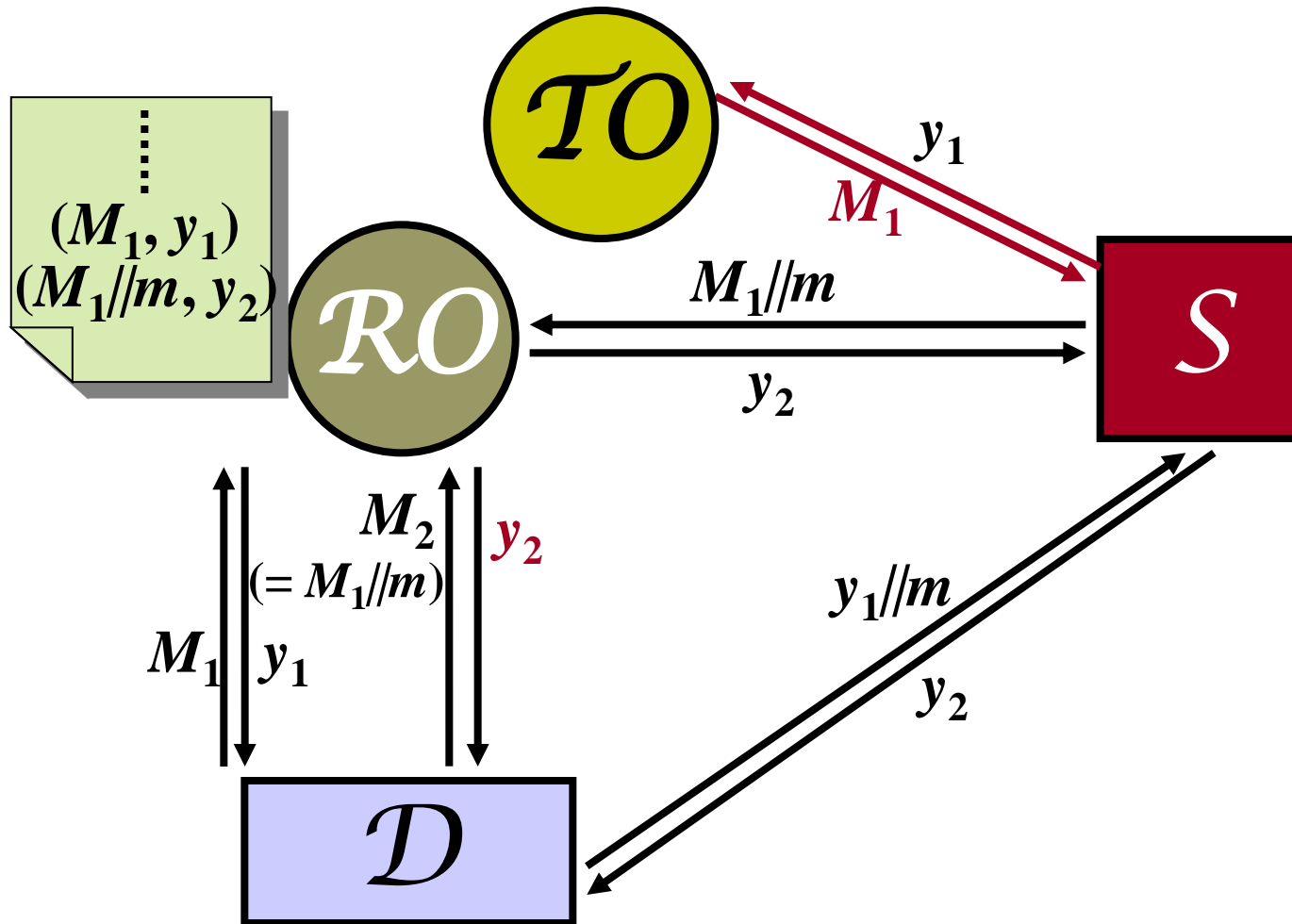
- **OAEP in LRO**

Adv uses private interface information to simulate decryption of OAEP, and then break IND-CCA!

# Traceable Random Oracle (TRO)



# Intuition of $MD \sqsubset TRO$

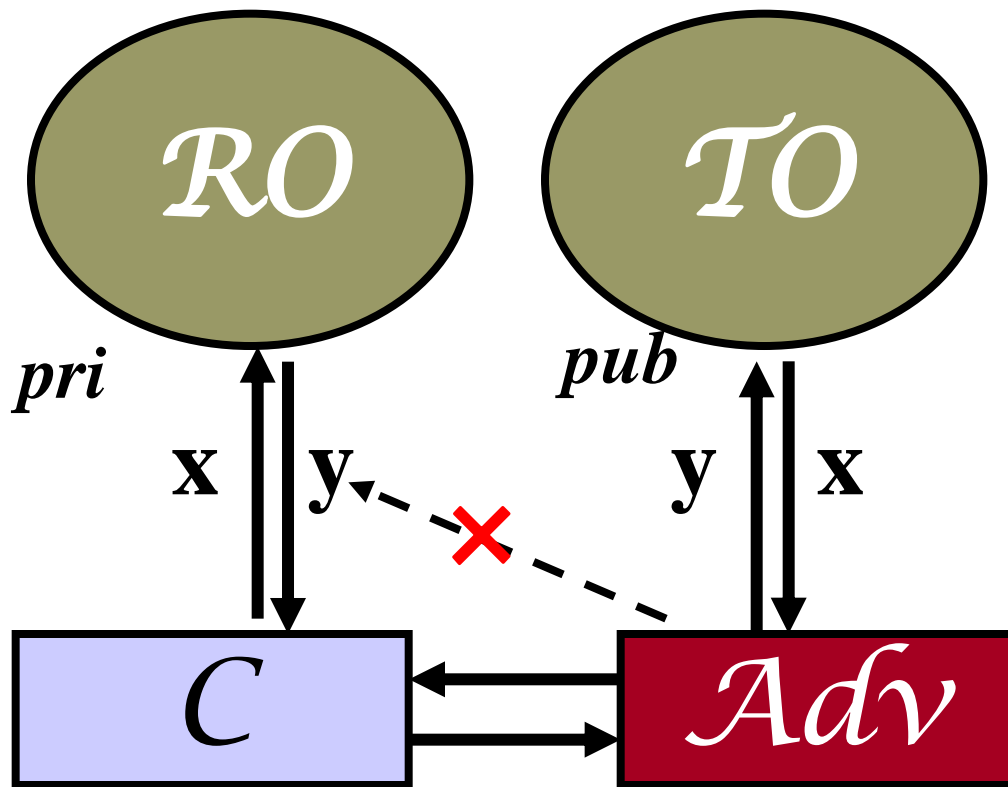


# Cryptosystems in TRO

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- **Secure: OAEP, ...**
  - OAEP is **insecure** in LRO.
  
- **Insecure: RSA-KEM, ...**
  - TRO requires **no leak** of **both query and response** in the private interface.

# Revisit Cryptosystem in TRO

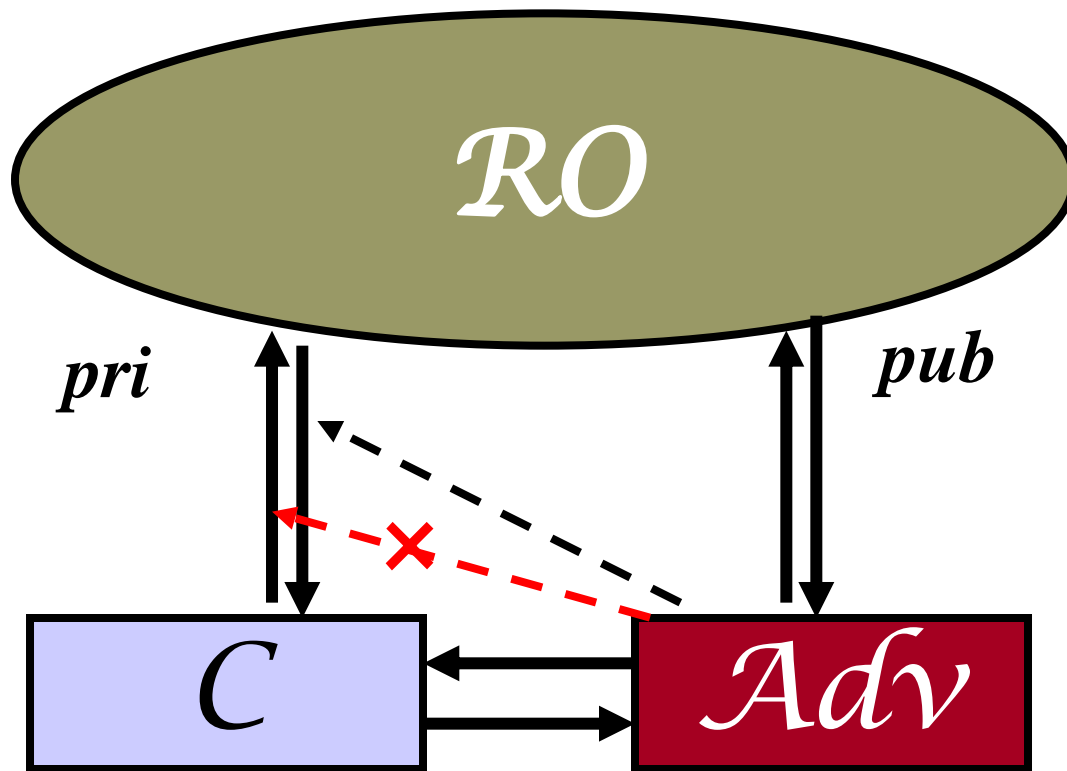


**TRO--->LRO**

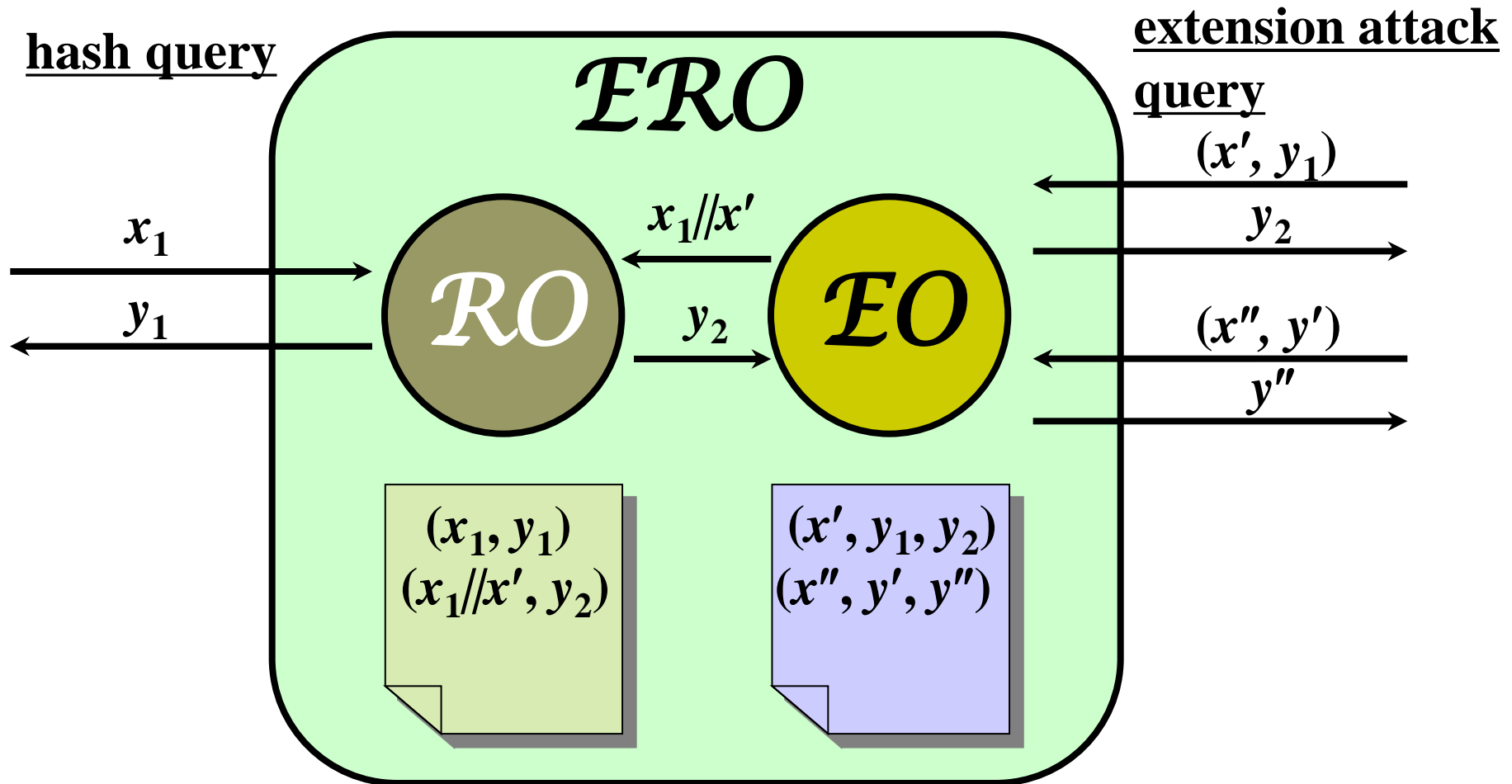


# RSA-KEM

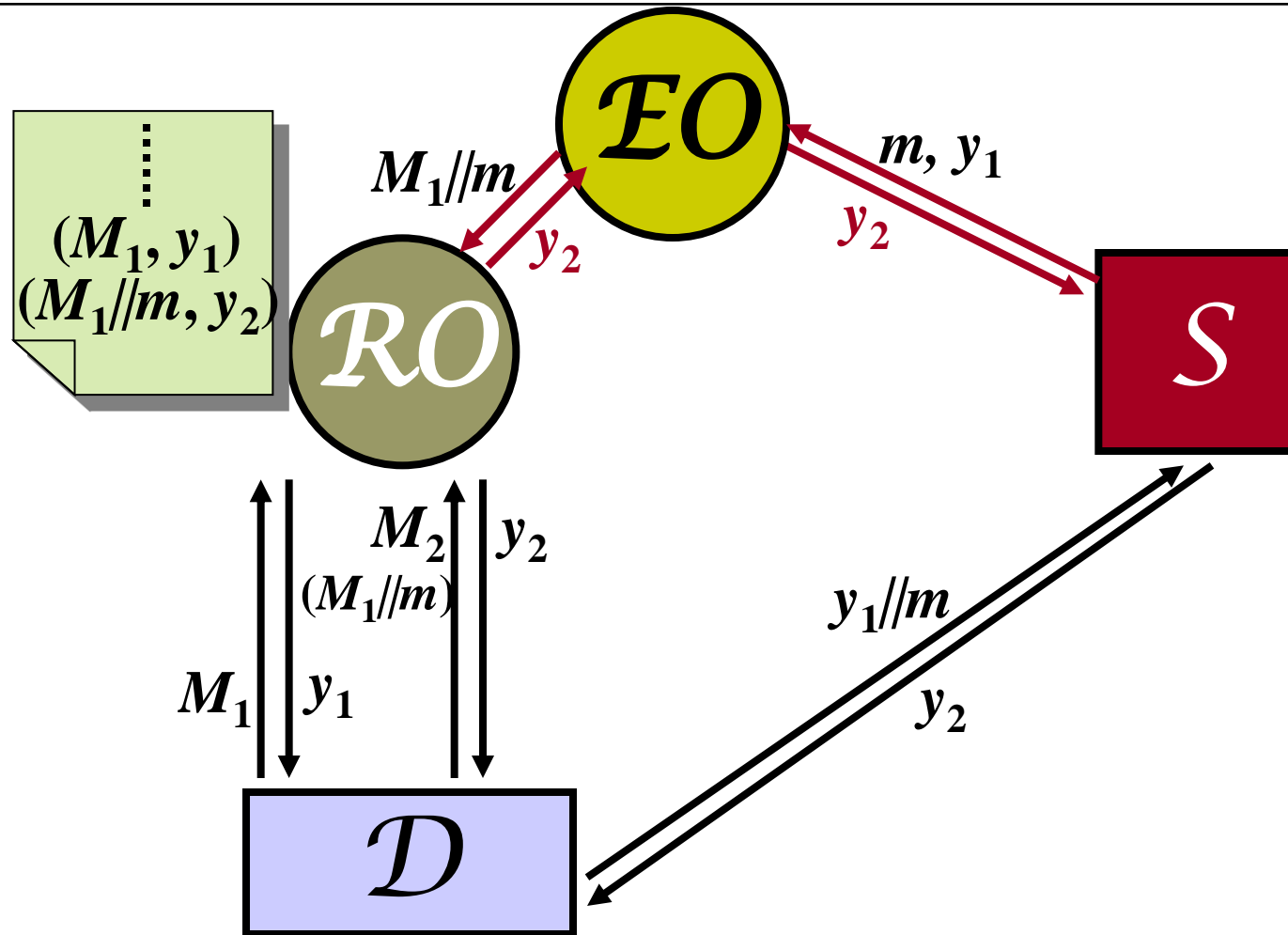
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# Extension Attack Simulatable Random Oracle (ERO)



# Intuition of $MD \sqsubseteq ERO$



# Cryptosystems in ERO

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- **Secure:** RSA-KEM, OAEP, FDH, ...
  - **RSA-KEM** is insecure in TRO and LRO model.
  
- **Insecure:** Secret-prefix MAC, ...
  - LEA **breaks EF-CMA** of Secret-prefix MAC in MD mode.

# Other Concerns

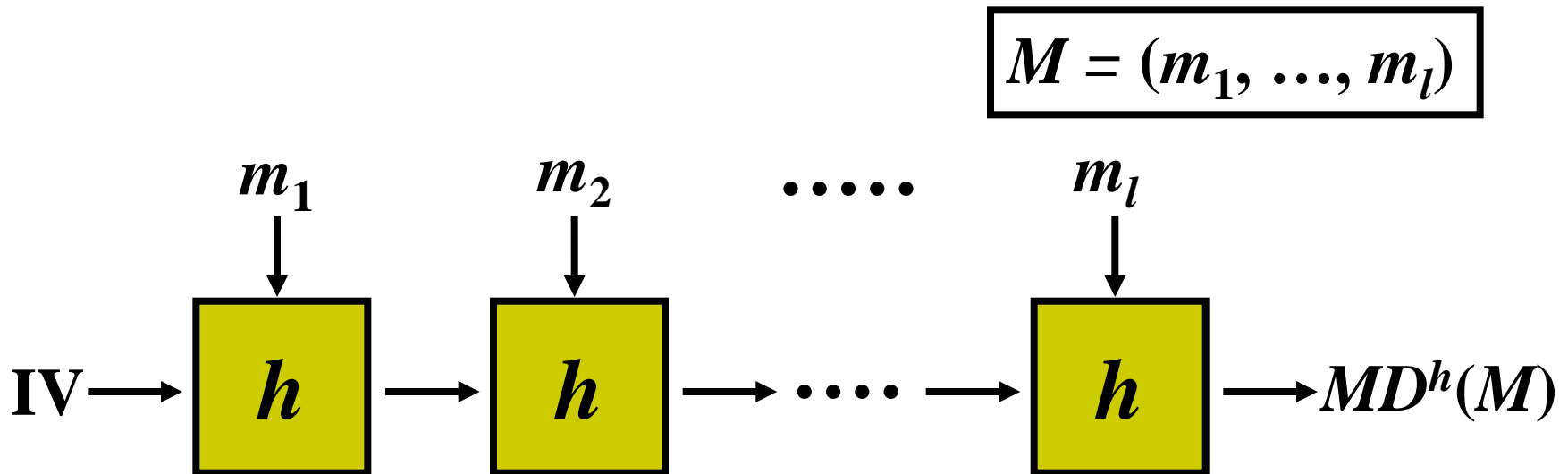
- **Compression function mode:**

**block-cipher-based**

- **Range extension:**

**Key derivation function (KDF)**

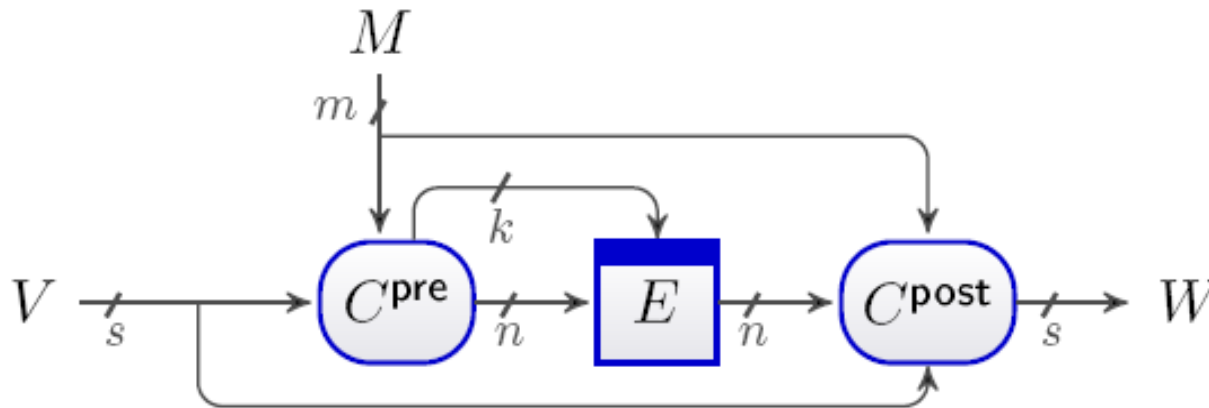
# Block-cipher-based MD Mode



Practical  $h$ : **block-cipher** based

Revisit cryptosystems in MD based on an  
**ideal block cipher**

# SCF: Stam's compression function



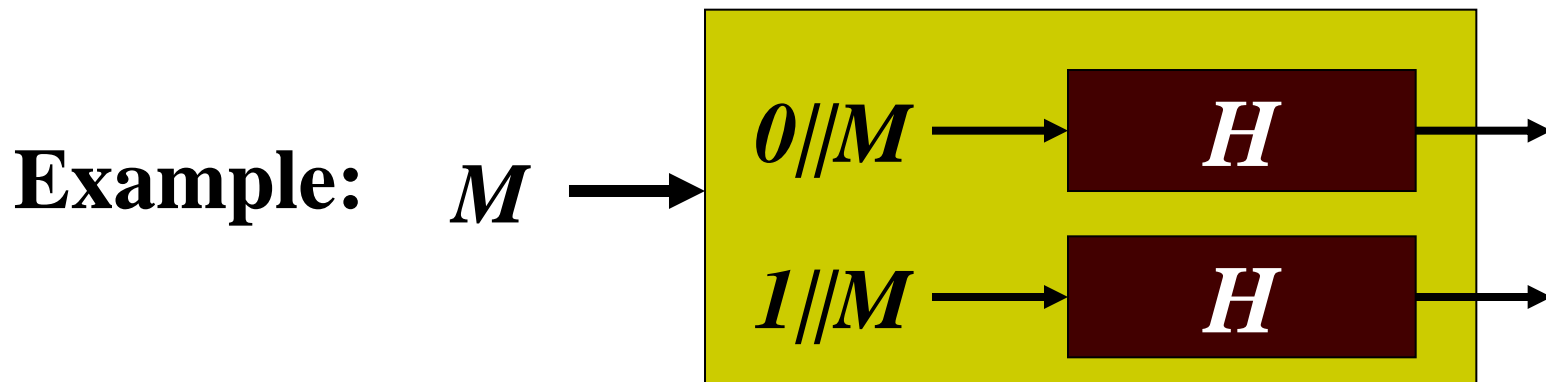
- $C^{\text{pre}}(\cdot)$  and  $C^{\text{post}}(\cdot)$  are public and deterministic functions.
- $E(\cdot, \cdot)$  is an **ideal** cipher.

*Stam, "Blockcipher-Based Hashing Revisited", FSE2009*

# KDF

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- Digests of stand-alone hash function are short.
  - RSA-FDH: at least 1024 bits.
  - SHA-2: at most 512 bits
- Parallel mode



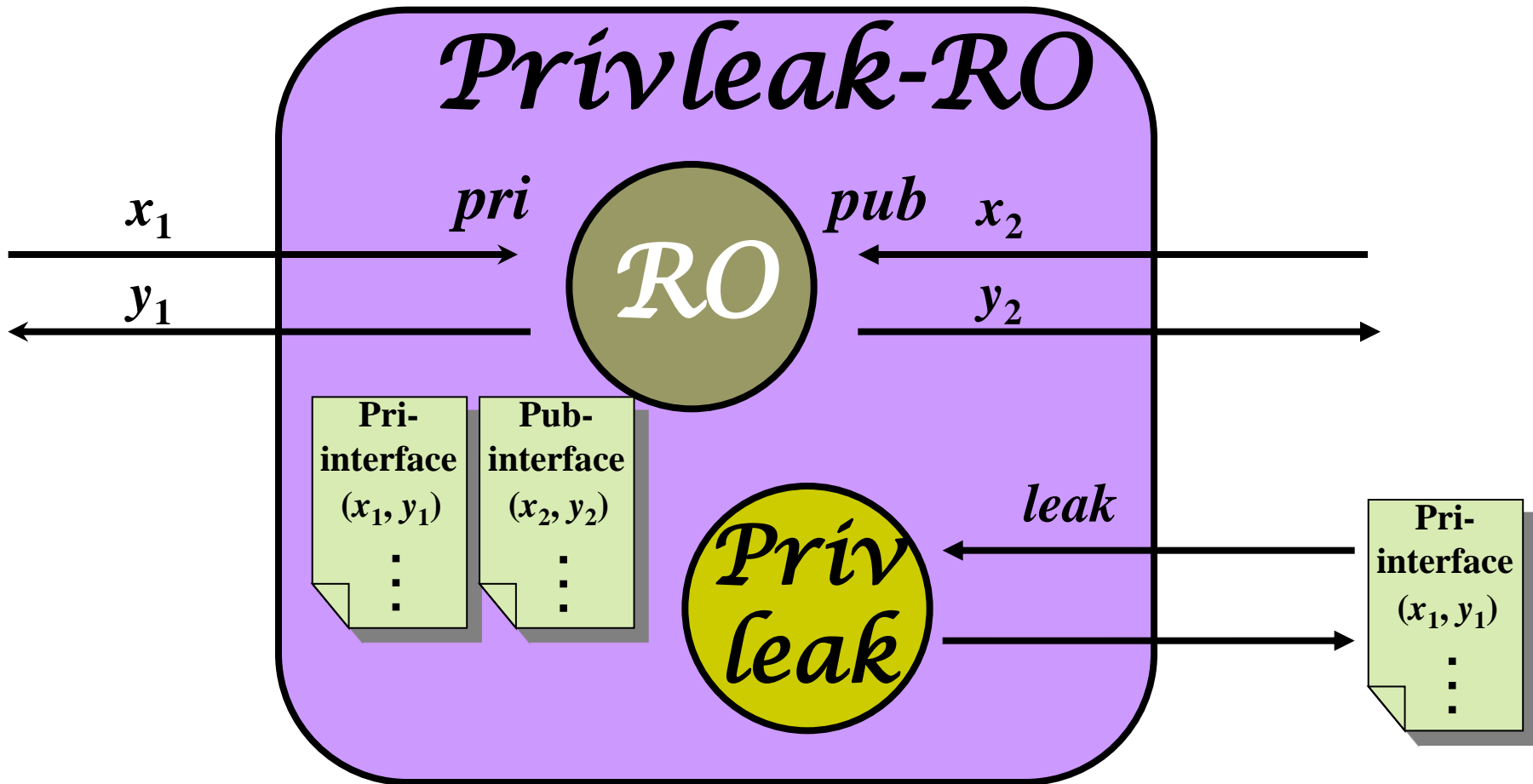


# Cryptosystems in KDF-MD

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- KDH, PSS, Fiat-Shamir, OAEP, RSA-KEM, PSEC-KEM, etc are **secure** in
  - KDF-MD based on FILRO
  - KDF-MD-SCFII (block-cipher-based).

# Privleak-RO



# Reason (brief)

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- KDF: parallel mode
  - On a query to one branch, simulator has to simulate all the other branches simultaneously.
  - Difference of hash lists will be used to distinguish KDF-MD from LRO!!!

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# Conclusion

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- Merkle-Damgård mode is able to guarantee the security of practical cryptosystems including FDH, OAEP, RSA-KEM etc.

**MD mode is still alive!!!**

Thank you!

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