Distributed Paillier Cryptosystem without Trusted Dealer

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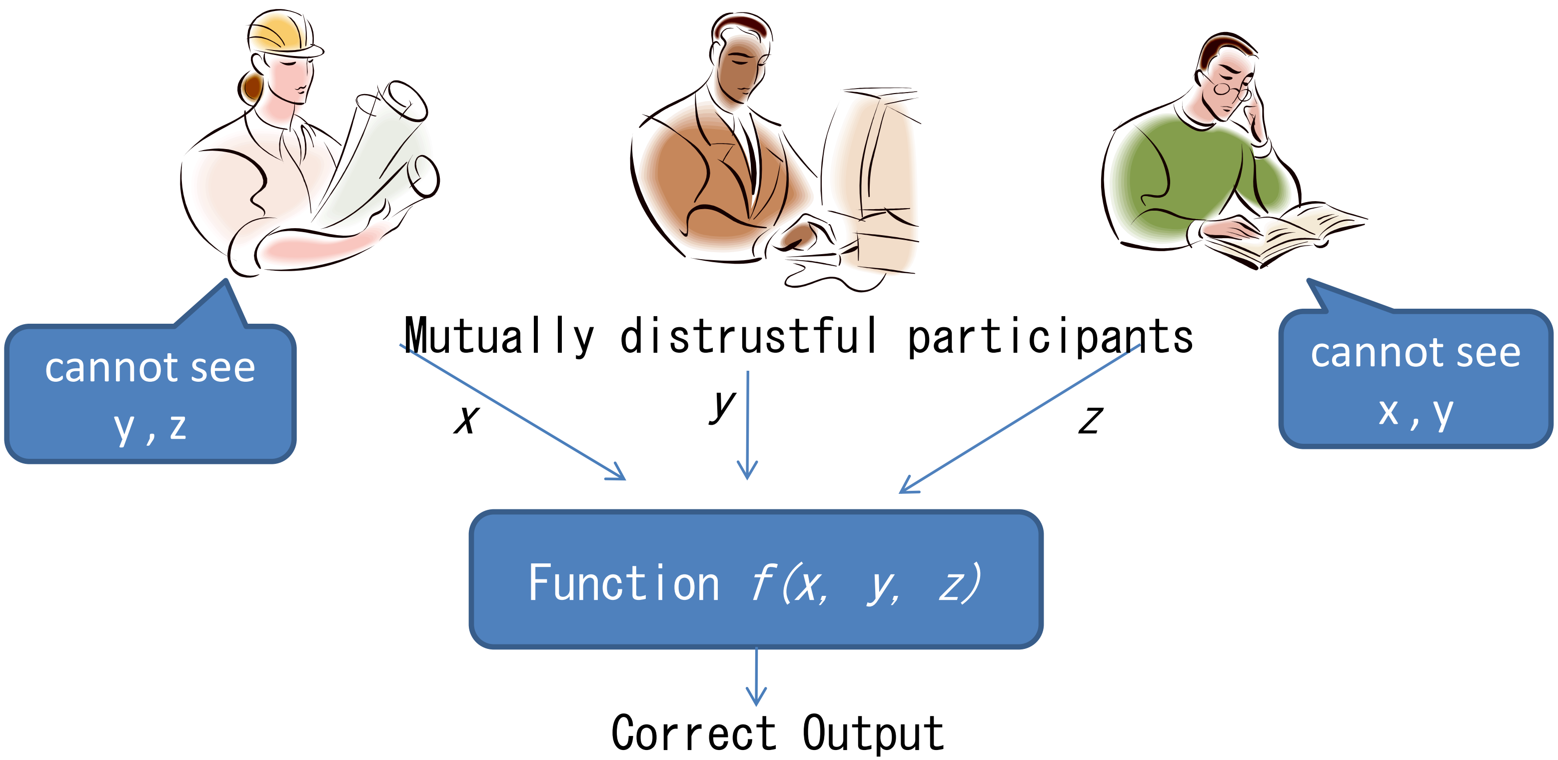
Multiparty Computation(MPC)



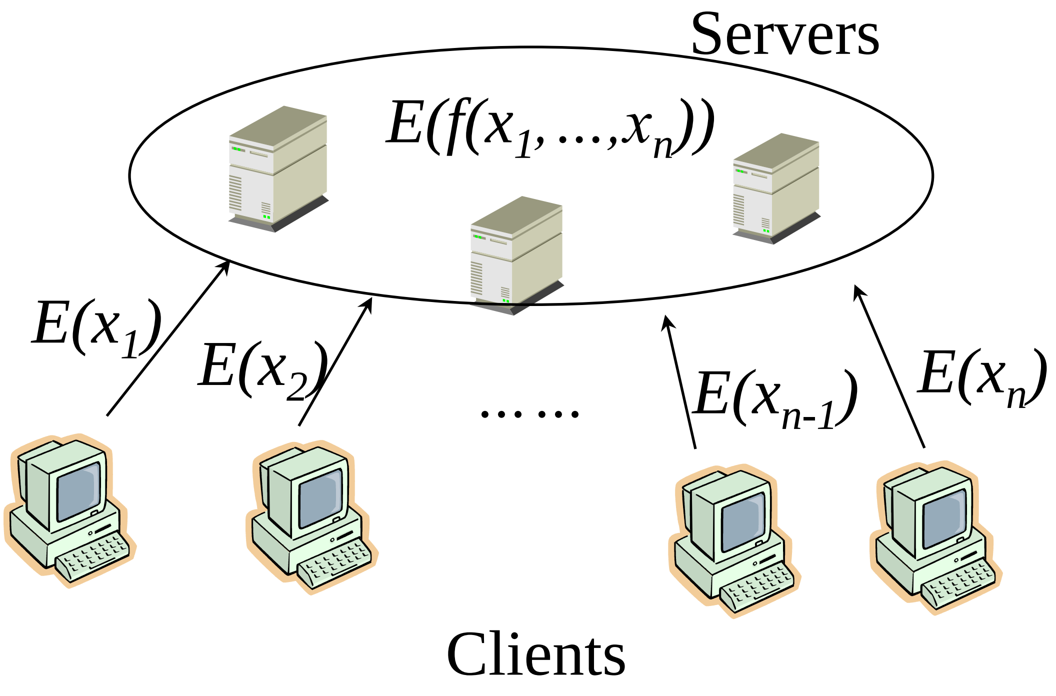
- Applications
  - Electronic Voting $f(x_1, \ldots, x_n) = \sum x_i$
  - Electronic Auction $f(x_1, \ldots, x_n) = \max(x_1, \ldots, x_n)$
  - Privacy Preserving Data Mining, etc

Two Major Approaches to MPC

- Shamir's Secret Sharing
  - Secrets are shared among the participants
- Threshold Homomorphic Cryptosystem (THC)
  - special public key cryptosystem
  - Secrets are encrypted
  - Homomorphic property
    - $E(m_1) * E(m_2) = E(m_1 + m_2)$
    - $E(m)^k = E(km)$

MPC Based on THC

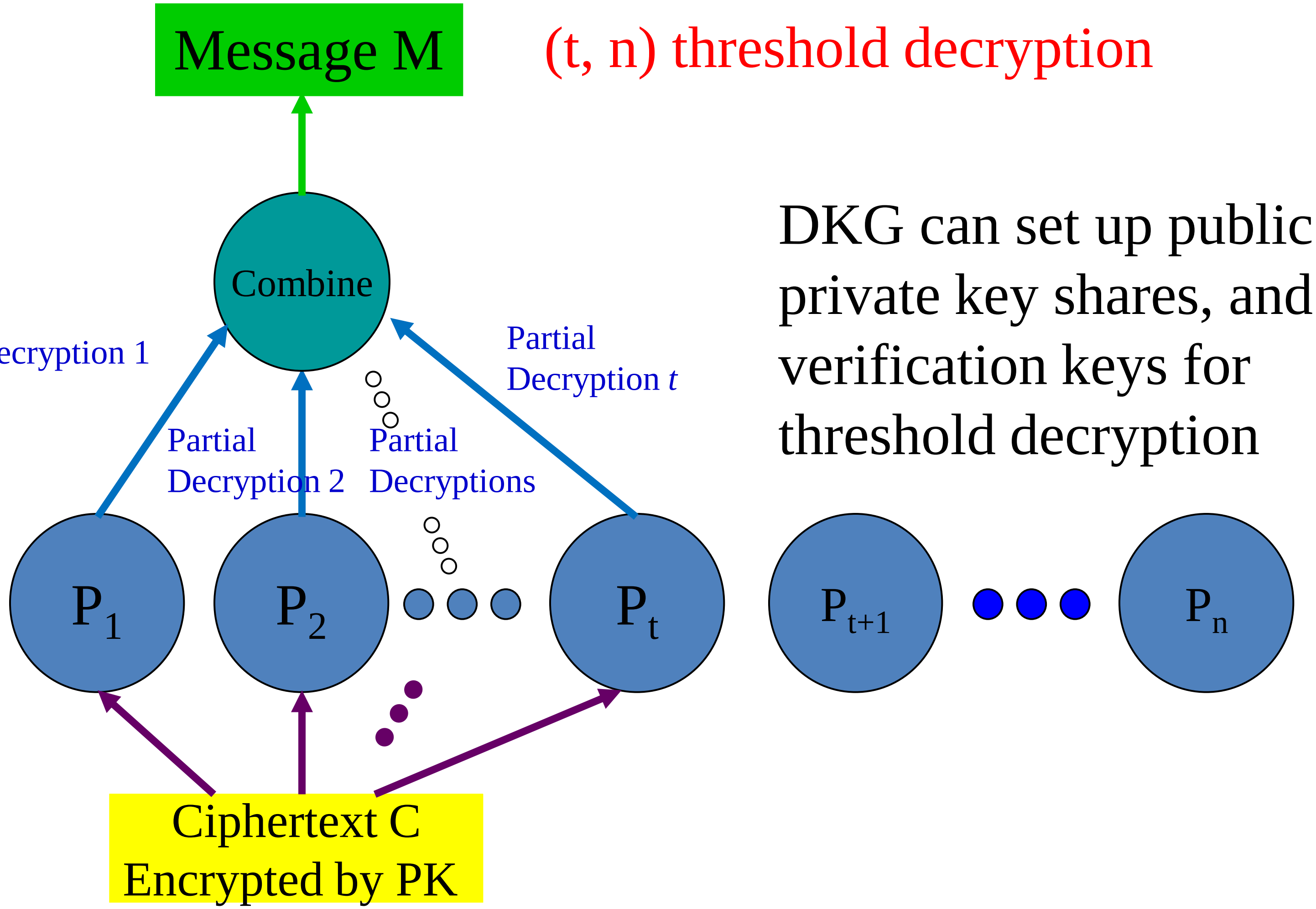
- Client-Server Model
  - Many clients provide encrypted data as inputs
  - Servers do blinded computation on encrypted data using homomorphic property



Initial Key Setup for THC

- In the initial key setup
  - the private decryption key must be shared among the participants
  - Verification keys must also be established for robustness against misbehaving participants
- The key setup can be done
  - by a trusted party
    - Single point of attack
  - by MPC again w/o trusted party (dealer)
    - Called Distributed Key Generation (DKG)

Key Setup & Threshold Decryption



(t, n) threshold decryption

DKG can set up public key, private key shares, and verification keys for threshold decryption

Popular Homomorphic Crypto

- ElGamal
  - Simple & robust DKG w/o trusted dealer
  - Additively homomorphic ElGamal can support only small plaintext space
- Paillier
  - Complex DKG or trusted dealer
    - Robust DKG w/o trusted setup (CRS) is non-trivial
  - Paillier can support huge plaintext space
  - Building block for many cryptographic protocols
    - meaningful to eliminate trusted dealer of private key to avoid a single point of attack

Related Work

- [BF97] realized first DKG for RSA in honest-but-curious model (i.e., non-robust)
  - Paillier cryptosystem also needs RSA modulus, so part of [BF97] can be used in DKG for Paillier
- [FMY98] extended [BF97] with robustness techniques
  - We use the different robustness techniques
  - The private key of Paillier is different from that of RSA , so we need to construct a different robust protocol

Related Work (Cont'd)

- [DK01] proposed threshold RSA signature using non-safe prime product with non-standard but reasonable assumption
  - We extend the assumption to Paillier setting to construct an efficient zero-knowledge proof for partial decryption share
- [DM10] proposed a novel distributed primality test in DKG for RSA
  - the protocol is designed only for three parties
  - needs a trusted setup (CRS for commitment)

Properties of Our Construction

- Based on [FPS00]
  - it assumes that a trusted dealer generates a safe safe prime product for RSA modulus
  - We do not need a safe prime product with additional assumption
- Robust protocol
- No trusted setup such as CRS
- Efficient ZKP for partial decryption share with non-binary challenge set
- Light range proof for shared secrets

Avoiding Safe Primes

- [FPS00] needs safe prime product where N = pq, p = 2p'+1, q = 2q'+1
  - But generating such N by DKG can be time-consuming though not impossible…
  - This condition is necessary for efficient proof of equality mod N
- We apply the assumption [DK01] to Paillier setting
  - Informally the assumption says that given N, p-1 (or q-1) includes a large prime factor Q such that it is infeasible to guess Q and 1/Q is negligible

Light Range Proof

- In [BF97], N is computed as
  - $N = (p_1 + p_2 + \ldots + p_n)(q_1 + q_2 + \ldots + q_n)$
  - p_i, q_i are chosen by participant P_i
- We need zero-knowledge proof that p_i, q_i are in the appropriate range $[2^{k-1}, 2^k - 1]$
  - classical bitwise range proof is inefficient for large numbers [Mao98]
  - [BCDG87] can be used with a group of known prime order where the expansion rate is 3, i.e., $p_i, q_i \in [0, 3*2^{k-1}]$

Sharing Private Key Robustly

- In our construction, $\varphi(N) = (p-1)(q-1)$ is shared over a prime field.
- the following values must be computed to share key
  - $\theta = \beta\varphi \bmod N$ revealed where β is a shared random secret
  - $\beta\varphi$ is shared over the integers
- We compute and reveal $\theta' = \beta\varphi + NR$ robustly
  - where R is a shared random secret over the integers
  - $\theta = \theta' \bmod N$
  - Sharing of $\beta\varphi$ obtained from sharing of $\theta' - NR$ over the integers where θ' and N are public values
  - can prove that θ' is indistinguishable from $\beta(N-1) + NR$
- Trial division on p,q can be done robustly in a similar way

Summary

- Constructed a distributed key generation protocol for Paillier cryptosystem based on [FPS00]
- DKG with Robustness
- No need to generate safe prime product
- No need for trusted setup
- Non-standard but reasonable assumption from [DK01] to realize efficient ZKP mod N

Thank you for you attention!